Design Optimization Methodology of On-Chip Spiral Inductor

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Abstract—This paper proposes an optimization methodology of on-chip spiral inductors. Characteristics of spiral inductors heavily depend on layout structures. We then formulate the characteristics with structural parameters and frequency as variables using a response surface method. The proposed method uses S-parameters to express inductor characteristics, and hence our method is independent of spiral geometries and equivalent circuit models. We explain the procedure of inductor optimization and demonstrate a design example.

I. INTRODUCTION

Radio-frequency integrated circuits (RF ICs) require high-quality on-chip inductors. Spiral inductors are one of the key components for a voltage-controlled oscillator (VCO), a low-noise amplifier (LNA), etc. [1, 2]. A spiral inductor is composed of metal lines traced spirally. At high frequency, several effects degrade the quality factor (Q) of the inductor, such as skin effect, eddy current, substrate loss, and self resonance [1]. These effects heavily depend on the layout structure of the spiral inductor. Several structures are proposed to obtain a high-Q inductor [3]. Thus spiral inductor design is one of the major interests for RF circuit designers.

We have to find the structure of high-Q spiral inductor that satisfies the required inductance value. One method to optimize the layout structure is a trial-and-error approach by measurements or simulations. However, this approach needs to repeat many manufactures or simulations, and hence it requires a long time until we obtain an optimal structure. The other approach is an analytic equation approach [4–6]. With the given equation, we can derive an optimal structure easily. However, we have to develop a new equation model for each technology, spiral structure and equivalent circuit model. Moreover, it is difficult to build the model that can express the characteristics for a wide frequency range.

In this paper, we propose a design methodology to obtain an optimal layout structure systematically. Our methodology uses a response surface method (RSM). We express the inductor characteristics by response surface functions of S-parameters. The response surface function is an equation with variables of physical layout parameters, such as line width, line space and number of turns. Exploiting the well-established theory of experimental design [7], the proposed method can reduce the number of manufactures or simulations efficiently. The proposed method builds the equation model in S-parameter domain, and hence our method is independent of equivalent circuit models and spiral geometries. The proposed method therefore can be applied to any types of inductors, and we can easily obtain various equivalent circuit models from the same S-parameter-based equations with little numerical calculation.

Section II explains required characteristics of spiral inductors and discusses conventional design methodologies to optimize the structure. We propose a design optimization methodology in Section III. Section IV presents the proposed optimization process using a design example. Section V describes the modeling accuracy of spiral inductors, and finally Section VI concludes this paper.

II. CHARACTERISTICS OF INDUCTOR AND DESIGN PROBLEMS

Spiral inductors are one of the most important passive devices for RF circuits. In this section, we explain the required characteristics of spiral inductors and the difficulties to obtain an optimal structure.

A. Overview of Spiral Inductors

Spiral inductors are very common devices to integrate inductors on silicon. A spiral inductor consists of metal lines that are traced spirally. Several spiral geometries with high-Q values are proposed, such as a rectangular (Fig. 1), an octagonal, a circular, a symmetry and etc. [1, 8]. Some equivalent-circuit models are also proposed. Figure 2 shows an example of the equivalent-circuit model. The equivalent-circuit model consists of several passive components; series metal resistance $R_s$, capacitance between metal and substrate $C_p$, and resistance of the silicon substrate $R_p$ in addition to inductance $L_s$. In this paper, we use the layout geometry shown in Fig. 1 and the equivalent circuit model in Fig. 2 as an example. The proposed method can cope with other layout geometries and equivalent circuit models with no theoretical limitations. The layout of spiral inductors can be determined by diameter ($d$), metal width ($w$), number of turns ($t$), space between lines ($d_g$), etc., and we call them “structural parameters.” Each component value of the equivalent circuit can be calculated from S-parameters with a straightforward translation.

One of the most important characteristics of inductors is the quality factor (Q), and it is defined by Eq. (1). The high-Q inductor means small loss [3].

$$Q = 2\pi \cdot \frac{\text{energy stored}}{\text{energy loss in one oscillation cycle}} \quad (1)$$

On-chip spiral inductors have the relatively high metal series resistance. This high resistivity is the main part of inductor loss. Also there are several effects that degrade Q at high frequency, e.g. skin effect, eddy current, substrate loss and self resonance [1]. These phenomena depend on layout patterns [3].
Therefore in order to obtain a high $Q$ inductor shortly, we need to estimate the inductor characteristics quickly and accurately against various physical parameters of inductor structures.

### B. Design Problem of Spiral Inductors

We usually optimize the inductor structure to maximize the $Q$ value of inductor, under the constraints of the demanded inductance value. We can naively optimize the structure with a lot of measurements and/or simulations by trial and error. However, this strategy does not have a guarantee that an optimal structure can be found in the limited design term. Moreover, without a smart guidance, it usually requires impractical number of iterations and tends to fall into a local optimum.

When we know an analytical equation model for a spiral inductor, we can easily optimize the structure analytically or numerically. The analytical model basically aims to express component values ($L_s$, $R_s$, etc.) of equivalent circuits [4–6]. The equations are usually polynomials of structural parameters. There are two types of the analytic models; one is a physical model [4, 5] and the other is a numerical model [6]. The former is based on a physical analysis. The latter is obtained by curve fitting from the measured values. As the target frequency becomes high, it is getting harder to devise a physical model that can consider high frequency effects, such as skin effect and eddy current. Nevertheless we have to derive equations for every layout strategy and for every technology, which is a severe drawback of the physical model approach. As for the numerical model, we derive a specialized equation model for each equivalent circuit model. If circuit designers want to use other equivalent circuit model, we need to reconstruct equations that are unique to the new equivalent circuit model, which is inconvenient for circuit designers and exhausting for model developers.

### III. PROPOSED METHOD TO OPTIMIZE THE SPIRAL INDUCTOR

In this section, we propose a design methodology to optimize a spiral inductor. We characterize a spiral inductor in S-parameter domain using a response surface method (RSM). Inductance and $Q$ values are derived from S-parameters. With the modeling in S-parameter domain, circuit designers easily change the equivalent circuit model for their purpose, and any types of spiral geometries can be handled. We can also optimize spiral inductors with small computational cost because the inductor characteristics are expressed as polynomial equations. The model characterization can exploit the theory of experimental design [7], and hence the characterization can be performed systematically with few samples.

### A. Modeling by Response Surface Method

The proposed methodology uses the RSM to build a model from measured and/or simulated inductor characteristics, and we optimize the spiral inductor using the developed model.

Using RSM, we can construct a simple model that represents a relationship between structural parameters and S-parameters. The model is derived from measured and/or simulated sample data. Figure 3 shows an example of a response surface function (RSF). The functions are the equations whose variables are structural parameters and frequency. The design constraint and objective functions are uniquely determined from the functions of S-parameters, when the equivalent circuit model is given. The constraint conditions are, for example, inductance value, the maximum parasitic capacitance or the maximum layout area. The objective function is usually $Q$ or parasitic capacitance, but it is not limited to them. These values are calculated from S-parameters. The component values of equivalent circuit can be also calculated from S-parameters. The transformation is dependent on the equivalent circuit model, but the derivation of its transformation is straightforward. We can therefore optimize the structure by the functions of S-parameters.
B. Advantage of Proposed Method

We can obtain the values of objective functions from the response surface functions for any structural parameters, whereas the response surface functions are constructed from discrete sample data, because the response surface method with smart experimental design can provide the continuous equation from limited sample data exploiting statistical theory. Using the objective function defined continuously, our method can systematically find an optimal structure analytically and/or numerically, which helps to reduce the optimization cost. The proposed method can include frequency uniformly as one of the variables in the response surface functions, and hence the proposed method can take the frequency-dependence of spiral inductors into consideration.

Our method can apply to any spiral geometries and any equivalent circuits, since we use S-parameters to characterize an inductor. If we use the analytical model discussed in Section II, we have to create a new model every time we change spiral geometries and/or equivalent circuit models. On the other hand, S-parameters are independent of the equivalent circuit model, and S-parameters can be measured to all the spiral geometries. Thanks to the modeling based on S-parameters, our method can be applied to any spiral geometries and any equivalent circuits in addition to Figs. 1 and 2.

C. Design Procedure of Optimization

The proposed method optimizes a spiral inductor using RSM as the following procedure.

1. Evaluate S-parameters of inductors by measurements and/or simulations.

2. Build the response surface functions of S-parameters from the sample data evaluated in Step 1.

3. Express constraint conditions and objective functions analytically or numerically using S-parameters.

4. Maximize/Minimize the objective function under the given constraints, and derive the optimal structure.

5. Calculate the component values of the equivalent circuit from S-parameters. Generate the inductor layout according to the obtained structural parameters.

In Step 1, we choose sample points of structural parameters, using the well-established technique of experimental design [7]. In Step 2, we build the response surface functions of S-parameters from the data evaluated in Step 1. The functions include the structural parameters and frequency as variables. The objective function is usually inductance or $Q$ and it is not S-parameters. The constraints are, for example, the inductance value, frequency, the maximum diameter or the maximum parasitic capacitance. They are not S-parameters, either. We then express the constraint and objective functions using S-parameters and structural parameters in Step 3. Step 4 finds the optimal structure that maximizes/minimizes the objective functions as far as the given constraints are satisfied. In Step 5, we generate the layout of the optimized inductor, and calculate the component values of the equivalent circuit from S-parameters.

<table>
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<tr>
<th>diameter[µm]</th>
<th>width[µm]</th>
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<tr>
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<td>2.5, 3.5, 4.5</td>
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Our method derives an optimal layout of a spiral inductor and its equivalent circuit model from several sample data. Both measurements and simulations provide S-parameters, and then it is reasonable to build response surface functions of S-parameters from them. The inductance and the $Q$ can be calculated from the response surface functions, and hence we can optimize the structure with small costs.

IV. DESIGN EXAMPLE

In this section, we demonstrate an optimization result of a spiral inductor in a 0.18µm process. We use a commercial 3D field-solver [9] to evaluate S-parameters. We maximize the $Q$ value of an inductor whose inductance is 3nH at the frequency of 2.5GHz.

A. Creating Sample Data

We use diameter ($d$), metal width ($w$), and number of turn ($t$) in Fig. 1 as structural parameters. The ranges of the parameters are as follows.

\[
150\mu m \leq d \leq 250\mu m
\]
\[
10\mu m \leq w \leq 20\mu m
\]
\[
t = 2.5, 3.5 \text{ and } 4.5
\]

Table I lists the twenty six (26) points of the sample data. The parameter sets of each sample point are determined by the experimental design. It is well known that the metal space ($s_p$) should be minimal to reduce fringing capacitance between metal lines [1]. Therefore, we use the minimum space of metal lines.

Table I The Sample Structures of Spiral Inductors for Creating Response Surface.

Compared with structural parameters, inductor characterization sweeping frequency by both measurement and simulation is much easier. We then sweep frequency from 2GHz to 3GHz by 0.1GHz for each sample point listed in Table I, and evaluate S-parameters.

\[
2\text{GHz} \leq f \leq 3\text{GHz}
\]

B. Creating the Response Surface Functions

The response surface model of S-parameters is built from the sample data in Table I. We use four response surface functions in
this example; real part and imaginary part of $s_{11}$ and $s_{21}$. Each function has four variables; diameter ($d$), metal width ($w$), number of turn ($t$) and frequency ($f$). The ranges of these parameters are shown in Eqs. (2)–(5) respectively.

In this example, we assume that the response surface functions are cubic functions of $d$, $w$, $t$ and $f$. The higher polynomials are also utilizable if you need. The functions are calculated from S-parameters evaluated by the electro-magnetic simulator. The response surfaces are shown in Fig. 4. A spiral structure cannot be realized in the following region, $d < 2(t \cdot w + (t - 1) \cdot s_p)$, shown by the broken line in Fig. 4(a). Figure 5 shows the response surface functions in frequency domain. The functions can express the frequency dependence of S-parameters. The accuracy of the response surface functions are discussed in Section V.

C. Calculating the Constraint Conditions and the Objective Functions

In this design example, the constraint condition is $L$ and the objective function is $Q$. They are calculated as follows:

$$Q = -\frac{\text{Im}(y_{11})}{\text{Re}(y_{11})}$$  \hfill (6)$$

$$L = -\frac{1}{2\pi f} \cdot \text{Im}\left(\frac{1}{y_{12}}\right)$$  \hfill (7)

$Y$-parameters can be calculated from S-parameters. 3-D plots of $L$ and $Q$ are shown in Fig. 6 and Fig. 7 respectively.

D. Optimization of the Structural Parameters

Figure 8 shows structures that satisfy inductance of $3 \pm 0.05\mu H$ at 2.5GHz. The $Q$ values of the structures are shown in Fig. 9. The curve with $t = 3.5$ realizes the maximum $Q$ value, so we select $t = 3.5$ and $d = 220\mu m$ as the optimal value. The optimal structures are derived from Figs. 8 and 9. The maximum value of $Q$ is 4.5 when the structure is $(d, w, t) = (220\mu m, 14\mu m, 3.5)$.

E. Creating the Layout and the Equivalent Circuit

The layout of spiral inductor is determined according to the structural parameters. The layout of the optimized inductor is shown in Fig. 10. The S-parameters of the optimal structure are determined from the response surface functions. The component values of the equivalent circuit are calculated from S-parameters. The equivalent circuit and the layout of the optimized inductor are shown in Fig. 11.

V. EVALUATION OF RESPONSE SURFACE FUNCTIONS

In this section, we examine the modeling accuracy of the response surface functions.

A. Accuracy at the Sample Points

We calculate the average and the maximum errors between the sample points and the response surface functions. The sample points listed in Table I are represented as the hollow circles in Fig. 12. The frequency range is 2–3 GHz. Table II shows the
Fig. 5. The sample data and the response surface functions of $\text{Re}(s_{11})$ vs. frequency at $t = 3.5$: (a) $w = 10\mu m$, (b) $d = 200\mu m$.

Fig. 6. 3-D plot of $L$ at $f = 2.5G\text{Hz}$ and $t = 3.5$.

Fig. 7. 3-D plot of $Q$ at $f = 2.5G\text{Hz}$ and $t = 3.5$.

Fig. 8. The structures which satisfy inductance are within $3\pm0.05\text{nH}$.

Fig. 9. $Q$ values of the structures shown in Fig. 8.

Fig. 10. The layout of the optimal spiral inductor.

Fig. 11. The equivalent circuit of the optimal spiral inductor.
results. The maximum errors of Re($s_{11}$) and Im($s_{11}$) are 10% or over. They are at the edge of the frequency range. The average errors of RSFs are less than 3%. Figure 13 shows comparisons between sample data and RSFs. All the sample data in frequency range 2–3GHz are plotted in Fig. 13. These results show that the response surface functions are relatively accurate at the sample points.

**B. Accuracy at Other Points**

The response surface functions are built from several sample points, so someone may doubt if the functions have excessively large errors except the sample points. We demonstrate the modeling accuracy at interpolating and extrapolating points in Fig. 12. The interpolating points are represented as squares. The frequency range is 2–3GHz. The structural parameters and the results are shown in Table III.

At the interpolating points, which corresponds to the shared area in Fig. 12, the errors of S-parameters, $L$ and $Q$ are lower than 4%. Our model is applicable with errors less than 4%, and it is accurate enough to be utilized for inductor optimization. In Fig. 12, the triangles means the extrapolating points. The errors of the extrapolating points are large as we expected. This is very natural because the response surface model does not aim to extrapolate sample data. When using RSM, we must carefully choose parameter ranges such that all the interested points are included. When the optimal structure is found in the extrapolating region, we should build new response surface functions adaptively from new sample points that includes all the points of interests.

**VI. Conclusion**

We propose a methodology to optimize a spiral inductor systematically. Our method is applicable to any spiral geometries and any equivalent-circuit models because of using S-parameters for modeling. The proposed method builds a response surface model of S-parameters with the structural parameters and frequency. The layout is determined by the structural parameters, and the equivalent-circuit model is calculated from the S-parameters. Therefore we can optimize inductor characteristics such as $Q$ using the derived polynomial functions of S-parameters.

We show the procedure to optimize a spiral inductor. The response surface functions are calculated from electro-magnetic simulations or measurement results. In our design experiment, the average error of the modeling function is less than 4%. We maximize $Q$ of the inductor with inductance of 3nH at 2.5GHz, and obtain the optimal inductor whose $Q$ is 4.5 at $(d, w, t) = (220\mu m, 14\mu m, 3.5)$.

**REFERENCES**


